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The NNLO gluon fusion Higgs production cross-section with many heavy quark

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Abstract: We consider extensions of the Standard Model with a number of additional heavy quarks which couple to the Higgs boson via top-like Yukawa interactions. We construct an effective theory valid for a Higgs boson mass which is lighter than twice the lightest heavy quark mass and compute the corresponding Wilson coefficient through NNLO. We present numerical results for the gluon fusion cross-section at the Tevatron for an extension of the Standard Model with a fourth generation of heavy quarks. The gluon fusion cross-section at NLO and in the approximation of infinitely heavy quarks is enhanced by a factor of 9 with respect to the Standard Model value. Tevatron experimental data can place stringent exclusion limits for the Higgs mass in this model. We find that the relative magnitude of the higher order QCD corrections with respect to the LO is similar to the Standard Model. Our result serves to carry out an exclusion analysis for a four-generation Standard Model with the same accuracy as in the Standard Model analysis.

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The NNLO gluon fusion Higgs production cross-section with many heavy quarks.

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ABSTRACT: We consider extensions of the Standard Model with a number of additional heavy quarks which couple to the Higgs boson via top-like Yukawa interactions. We construct an effective theory valid for a Higgs boson mass which is lighter than twice the lightest heavy quark mass and compute the corresponding Wilson coefficient through NNLO. We present numerical results for the gluon fusion cross-section at the Tevatron for an extension of the Standard Model with a fourth generation of heavy quarks. The gluon fusion cross-section is enhanced by a factor of roughly 9 with respect to the Standard Model value. Tevatron experimental data can place stringent exclusion limits for the Higgs mass in this model.

1. Introduction

The discovery of the Higgs boson will introduce a new era in particle physics. The long standing theoretical problem of understanding the mechanism for electroweak symmetry breaking will be tackled for the first time using direct experimental findings. The measurement of the Higgs boson mass and the production cross-sections of its various signatures will be important constraints in formulating a theory of particle interactions at high energies.

The interaction of the Higgs boson and gluons is particularly important at hadron collider experiments. In the Standard Model (SM), the gluon fusion cross-section is the largest among all production cross-sections. The LHC will be able to discover the SM Higgs boson in this production channel for the full range of its allowed mass values. The branching ratios for the decays of the SM Higgs boson are dominated by other Higgs boson interactions which involve the bottom quark and electroweak gauge bosons. However, the Higgs-gluon interaction is still not negligible; a fraction of up to about 7% of Higgs bosons may decay to gluons, depending on the Higgs boson mass.

Given that gluons are massless, a Higgs-gluon interaction arises as a loop effect via other massive coloured particles which couple to the Higgs boson. Physics beyond the Standard Model can alter significantly the strength of this interaction in various ways. One possibility is that new coloured particles are not much heavier than the top quark, and their contribution is therefore not suppressed. A second possibility is that new coloured particles may be heavier than the top-quark but they have an enhanced Yukawa coupling to the Higgs. A third possibility is that such particles are quite heavier than the top-quark, but their multiplicity is large, thus building up a significant cumulative contribution.

A Higgs boson is often assumed to be lighter than about twice the mass of the top-quark and twice the mass of new undiscovered particles which are hypothesized in extensions of the Standard Model. Light new particles are hard to accommodate given the vigorous experimental searches for new physics at LEP and the Tevatron. On the other hand, they cannot be very heavy or, alternatively, they must have a rather strong Higgs coupling if they contribute in reducing the fine tuning of the Higgs mass. Therefore, it is important to calculate their contribution in the gluon fusion process as well as in the decay of a Higgs boson to gluons.

Assuming a Higgs boson which is lighter than production thresholds of new heavy particles, we can factorize the effect of new physics and QCD in the process $gg \rightarrow H$, by means of an effective field theory where the top-quark and all other possible heavy coloured states which couple to the Higgs boson are integrated out. The effect of these heavy particles is included in the Wilson coefficients of an effective theory with operators of the Higgs boson and light quarks and gluons.

The possibilities for viable extensions of the Standard Model which alter the Higgs-gluon interaction are many, and an equal number of matching calculations is required for their study. This is a rather easy task at leading order in the strong coupling. However, experience from the Standard Model shows that a precise estimate of the gluon-fusion cross-section and the Higgs decay width to gluons requires calculations through next-to-next-to-leading-order (NNLO) in the strong coupling.

In this paper, we consider extensions of the Standard Model with additional heavy quarks. We assume that these quarks have a Higgs Yukawa interaction of the same type as Standard Model quarks. The existence of such quarks has dramatic implications for the Higgs production cross-section in gluon fusion. At leading order, and in the limit where the heavy quarks are much heavier than half the mass of the Higgs boson, the cross-section scales as n_h^2 , where n_h is the number of heavy quarks. Current measurements at the Tevatron [1] and early data from the LHC can therefore constrain severely such models. We first construct an effective Lagrangian integrating out the top-quark and the additional heavy quarks. We compute the Wilson coefficient of the Higgs-gluon effective interaction through NNLO in the strong coupling expansion. Finally, we present numerical results for the gluon fusion cross-section at the Tevatron in a specific model with a fourth quark generation.

2. The effective Lagrangian

We consider an arbitrary extension of the SM through new heavy quarks transforming under the fundamental representation of the QCD gauge group $SU(3)$. The number of heavy quarks, including the top, is n_h . We will denote their mass by m_q , with $q = 1 \dots n_h$. We assume that the new quarks, as the SM top, couple to the Higgs boson H through their mass. Therefore, the Lagrangian we begin with is

$$\mathcal{L} = \mathcal{L}_{QCD}^{n_l} + \sum_{q=1}^{n_h} \bar{\psi}_q (i\not{D} - m_q) \psi_q + \mathcal{L}_Y \quad , \quad \mathcal{L}_Y = -\frac{H}{v} \sum_{q=1}^{n_h} m_q \bar{\psi}_q \psi_q \quad . \quad (2.1)$$

Here D_μ is the covariant derivative in the fundamental representation and $\mathcal{L}_{QCD}^{n_l}$ is the QCD Lagrangian with only the n_l flavours of light quarks. We take these quarks to be massless.

We focus on the changes that the heavy quarks induce on the Higgs production through gluon fusion. When the quarks that couple to the Higgs boson are heavier than half the Higgs boson mass, we can integrate them out. In this limit, we can replace the original Lagrangian (2.1) with an effective Lagrangian

$$\mathcal{L}^{eff} = \mathcal{L}_{QCD}^{eff, n_l} - C_1 \frac{H}{v} \mathcal{O}_1 \quad . \quad (2.2)$$

C_1 is the Wilson coefficient [2] relative to the only dimension-four local operator \mathcal{O}_1 that arises when we integrate out the heavy quarks and all the quarks remaining are massless [3],

$$\mathcal{O}_1 = \frac{1}{4} G_{\mu\nu}^a G'^{a\mu\nu} \quad . \quad (2.3)$$

In this expression, $G_{\mu\nu}^a$ is the field strength tensor in the effective theory. In Eq. (2.2), $\mathcal{L}_{QCD}^{eff, n_l}$ describes the interactions among light quarks. It has the same form as $\mathcal{L}_{QCD}^{n_l}$, but with different parameters and field normalizations because of the contributions from heavy quarks loops. We relate the parameters in the effective theory to the parameters in the full theory through multiplicative decoupling constants ζ_i . We will denote quantities in

the effective theory with a prime. The derivation of the decoupling constants is reviewed in [4]. In Section 4, we describe the main steps of their calculation and give the relevant results.

3. Method

We compute the Wilson coefficient C_1 up to three loops. Diagrams containing both the heavy mass scales appear for the first time at the three-loop order. We start from the bare amplitude $\mathcal{M}_{gg \rightarrow H}^0$ for the process $gg \rightarrow H$ in the full theory,

$$\mathcal{M}_{gg \rightarrow H}^0 \equiv \mathcal{M}_{\mu_1 \mu_2}^{0, a_1 a_2}(p_1, p_2) \epsilon_{a_1}^{\mu_1} \epsilon_{a_2}^{\mu_2} . \quad (3.1)$$

Here, p_1 and p_2 are the momenta of the two gluons with polarizations $\epsilon_{a_1}^{\mu_1}$ and $\epsilon_{a_2}^{\mu_2}$. This amplitude is related to the bare Wilson coefficient C_1^0 by [4]

$$\frac{\zeta_3^0 C_1^0}{v} = \frac{\delta^{a_1 a_2} (g^{\mu_1 \mu_2} (p_1 \cdot p_2) - p_1^{\mu_2} p_2^{\mu_1})}{(N^2 - 1)(d - 2)(p_1 \cdot p_2)^2} \mathcal{M}_{\mu_1 \mu_2}^{0, a_1 a_2}(p_1, p_2) \Big|_{p_1=p_2=0} . \quad (3.2)$$

N is the number of colours and $d = 4 - 2\epsilon$ is the dimension of space-time. Bare quantities are denoted by the superscript “0”. The factor ζ_3^0 is the bare decoupling coefficient by which the bare gluon field $G_\mu^{0, a}$ is rescaled in the effective theory,

$$G_\mu^{0, a} = \sqrt{\zeta_3^0} G_\mu^{0, a} . \quad (3.3)$$

We generate the Feynman diagrams \mathcal{F} for the amplitude through three loops using QGRAF [5]. We then perform an expansion of all diagrams in the external momenta p_1, p_2 , by applying the following differential operator [6] to their integrand:

$$\mathcal{D}\mathcal{F} = \sum_{n=0}^{\infty} (p_1 \cdot p_2)^n [\mathcal{D}_n \mathcal{F}]_{p_1=p_2=0} , \quad (3.4)$$

with

$$\mathcal{D}_0 = 1, \quad \mathcal{D}_1 = \frac{1}{d} \square_{12}, \quad \mathcal{D}_2 = -\frac{1}{2(d-1)d(d+2)} \{ \square_{11} \square_{22} - d \square_{12}^2 \} , \quad (3.5)$$

and $\square_{ij} \equiv g^{\mu\nu} \frac{\partial^2}{\partial p_i^\mu \partial p_j^\nu}$.

Differential operators of higher orders are not needed for the expansion in the external momenta at leading order.

After Taylor expansion, all the Feynman diagrams are expressed in terms of one-, two- and three-loop vacuum bubbles by using linear transformations on the loop-momenta k_i :

$$I_1 [\nu_1] \equiv \int \frac{d^d k_1}{i\pi^{d/2}} \frac{1}{\mathcal{P}_1^{\nu_1}} , \quad (3.6)$$

$$I_2 [\nu_1, \nu_2, \nu_5] \equiv \int \frac{d^d k_1 d^d k_2}{(i\pi^{d/2})^2} \frac{1}{\mathcal{P}_1^{\nu_1} \mathcal{P}_2^{\nu_2} \mathcal{P}_5^{\nu_5}} , \quad (3.7)$$

$$I_{3a} [\nu_1, \nu_2, \nu_3, \nu_5, \nu_6, \nu_7] \equiv \int \frac{d^d k_1 d^d k_2 d^d k_3}{(i\pi^{d/2})^3} \frac{1}{\mathcal{P}_1^{\nu_1} \mathcal{P}_2^{\nu_2} \mathcal{P}_3^{\nu_3} \mathcal{P}_5^{\nu_5} \mathcal{P}_6^{\nu_6} \mathcal{P}_7^{\nu_7}} , \quad (3.8)$$

$$I_{3b} [\nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6] \equiv \int \frac{d^d k_1 d^d k_2 d^d k_3}{(i\pi^{d/2})^3} \frac{1}{\mathcal{P}_1^{\nu_1} \mathcal{P}_2^{\nu_2} \mathcal{P}_3^{\nu_3} \mathcal{P}_4^{\nu_4} \mathcal{P}_5^{\nu_5} \mathcal{P}_6^{\nu_6}} , \quad (3.9)$$

$$I_{3c} [\nu_1, \tilde{\nu}_2, \tilde{\nu}_3, \nu_4, \nu_5, \nu_6] \equiv \int \frac{d^d k_1 d^d k_2 d^d k_3}{(i\pi^{d/2})^3} \frac{1}{\mathcal{P}_1^{\nu_1} \tilde{\mathcal{P}}_2^{\tilde{\nu}_2} \tilde{\mathcal{P}}_3^{\tilde{\nu}_3} \mathcal{P}_4^{\nu_4} \mathcal{P}_5^{\nu_5} \mathcal{P}_6^{\nu_6}} , \quad (3.10)$$

with

$$\begin{aligned} \mathcal{P}_1 &= k_1^2 - m_q^2 , \\ \mathcal{P}_2 &= k_2^2 - m_q^2 , & \tilde{\mathcal{P}}_2 &= k_2^2 - m_{q'}^2 , \\ \mathcal{P}_3 &= k_3^2 - m_q^2 , & \tilde{\mathcal{P}}_3 &= k_3^2 - m_{q'}^2 , \\ \mathcal{P}_4 &= (k_1 - k_2 + k_3)^2 - m_q^2 , \\ \mathcal{P}_5 &= (k_1 - k_2)^2 , \\ \mathcal{P}_6 &= (k_2 - k_3)^2 , \\ \mathcal{P}_7 &= (k_3 - k_1)^2 , \end{aligned} \quad (3.11)$$

and $\nu_i, \tilde{\nu}_i$ positive or negative integers. The third three-loop vacuum bubble I_{3c} contains two heavy quarks of different mass, m_q and $m_{q'}$.

We perform a reduction of the above integral topologies to master integrals using the algorithm of Laporta [7] and the program AIR [8]. We find five master integrals,

$$\begin{aligned} \mathcal{I}_1 &= I_1[1] \\ &= - (m_q^2)^{1-\epsilon} \Gamma(-1+\epsilon) , \end{aligned} \quad (3.12)$$

$$\begin{aligned} \mathcal{I}_2 &= I_{3a}[1, 0, 1, 1, 1, 0] \\ &= (m_q^2)^{2-3\epsilon} \frac{\Gamma^2(1-\epsilon) \Gamma(\epsilon) \Gamma^2(-1+2\epsilon) \Gamma(-2+3\epsilon)}{\Gamma(2-\epsilon) \Gamma(-2+4\epsilon)} , \end{aligned} \quad (3.13)$$

$$\mathcal{I}_3 = I_{3b}[1, 1, 1, 1, 0, 0] , \quad (3.14)$$

$$\mathcal{I}_4 = I_{3c}[1, 1, 1, 1, 0, 0] , \quad (3.15)$$

$$\mathcal{I}_5 = I_{3c}[2, 1, 1, 1, 0, 0] . \quad (3.16)$$

Single-scale master integrals appear in the calculation of the SM Wilson coefficient [9, 10] and can be computed with MATAD [11]. For the remaining two-scale master integrals we used result from [12]. We checked all the master integrals independently through sector decomposition with the program of Ref. [13].

After these steps, the RHS of Eq. (3.2) becomes

$$\begin{aligned}
\frac{\zeta_3^0 C_1^0}{v} = & \sum_{q=1}^{n_h} \left\{ \frac{1}{3} \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right) \left[-1 + \epsilon \left[1 + 2 \log(m_q^0) \right] \right. \right. \\
& - 2\epsilon^2 \left[\log^2(m_q^0) + \log(m_q^0) + \frac{\pi^2}{24} \right] + \mathcal{O}(\epsilon^3) \Big] \\
& + \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right)^2 \left[-\frac{1}{4} + \epsilon \left[\log(m_q^0) + \frac{31}{36} \right] + \mathcal{O}(\epsilon^2) \right] \\
& + \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right)^3 \left[-\frac{1}{32\epsilon^2} + \frac{1}{\epsilon} \left[\frac{3 \log(m_q^0)}{16} - \frac{223}{576} \right] \right. \\
& \quad \left. + n_l \left[-\frac{5}{144} \frac{1}{\epsilon} + \frac{103}{864} + \frac{5 \log(m_q^0)}{24} \right] \right. \\
& \quad \left. - \frac{9}{16} \log^2(m_q^0) + \frac{223 \log(m_q^0)}{96} - \frac{\pi^2}{128} + \frac{5975}{3456} + \mathcal{O}(\epsilon) \right] \Big\} \\
& - \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right)^3 \sum_{q>q'} \left\{ \frac{1}{16\epsilon^2} - \frac{1}{16\epsilon} \left[3 (\log(m_q^0) + \log(m_{q'}^0)) + \frac{89}{18} \right] + \frac{1}{2} \log(m_q^0) \log(m_{q'}^0) \right. \\
& \quad \left. + \frac{89}{96} [\log(m_q^0) + \log(m_{q'}^0)] + \frac{5}{16} [\log^2(m_q^0) + \log^2(m_{q'}^0)] + \frac{1051 + 27\pi^2}{1728} + \mathcal{O}(\epsilon) \right\} .
\end{aligned} \tag{3.17}$$

The first sum in this expression runs over all single-scale diagrams and corresponds to n_h copies of the SM Wilson coefficient. The second sum accounts for the 3-loop diagrams in which either of the two massive quarks couples to the Higgs boson. We already symmetrized it over q, q' . In Eq. (3.17) we introduced the factor

$$S_\epsilon = e^{-\epsilon \gamma_E} (4\pi)^\epsilon . \tag{3.18}$$

4. Decoupling and renormalization

The RHS of Eq. (3.17) contains the bare masses of the heavy quarks m_q^0 and the bare coupling constant α_s^0 in the full theory; $C_1^0 = C_1^0(\alpha_s^0, m_q^0)$. The bare strong coupling in the full theory is related to the bare strong coupling in the effective theory $\alpha_s'^0$ by the decoupling constants ζ_g^0 [4, 10],

$$\alpha_s'^0 = (\zeta_g^0)^2 \alpha_s^0 . \tag{4.1}$$

Similiarly,

$$\alpha_s' = (\zeta_g)^2 \alpha_s . \tag{4.2}$$

Using these relations, we obtain the bare Wilson coefficient as a function of the bare parameters in the effective theory and of the bare mass of the heavy quarks in the full theory, $C_1^0 = C_1^0(\alpha_s'^0, m_q^0)$. The bare parameters are related to the renormalized ones

through multiplicative renormalization constants Z_i as

$$\alpha_s'^0 = \mu^{2\epsilon} Z'_\alpha \alpha'_s(\mu) \quad , \quad (4.3)$$

$$\alpha_s^0 = \mu^{2\epsilon} Z_\alpha \alpha_s(\mu) \quad , \quad m_q^0 = Z_{m_q} m_q(\mu) \quad . \quad (4.4)$$

All the parameters in Eq. (4.3) are in the effective theory and all the parameters in Eq. (4.4) are in the full theory. Finally, we renormalize the Wilson coefficient itself through a renormalization factor Z_{11} [3, 14, 15],

$$C_1 = \frac{1}{Z_{11}} C_1^0 \quad . \quad (4.5)$$

4.1 Details of the calculation

A convenient way to compute the gluon field decoupling ζ_3^0 is through the relation

$$\zeta_3^0 = 1 + \Pi_G^0(p=0) \quad , \quad (4.6)$$

where Π_G^0 is the transverse component of the gluon self-energy in the full theory. This quantity is computed at zero external momentum. Since we work in dimensional regularization, only diagrams containing at least one massive quark contribute. We find only one diagram per heavy flavour at one loop and seven at two loops. We employ the same calculation techniques as in Section 3. Our result reads

$$\begin{aligned} \zeta_3^0 = 1 + \sum_{q=1}^{n_h} \left\{ \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right) \left[\frac{1}{6\epsilon} - \frac{\log(m_q^0)}{3} + \epsilon \frac{\pi^2 + 24 \log^2(m_q^0)}{72} \right] \right. \\ \left. + \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right)^2 \left[\frac{3}{32\epsilon^2} - \frac{1 + 24 \log(m_q^0)}{64\epsilon} + \frac{3}{4} \log^2(m_q^0) + \frac{1}{16} \log(m_q^0) + \frac{91}{1152} + \frac{\pi^2}{64} \right] \right\} \quad . \end{aligned} \quad (4.7)$$

The bare decoupling parameter of the strong coupling constant, ζ_g^0 , can be computed as

$$\zeta_g^0 = \frac{\tilde{\zeta}_1^0}{\tilde{\zeta}_3^0 \sqrt{\zeta_3^0}} \quad , \quad (4.8)$$

where $\tilde{\zeta}_1^0$, $\tilde{\zeta}_3^0$ and ζ_3^0 are the bare decoupling constants of the gluon-ghost vertex, of the ghost field and of the gluon field respectively.

The bare decoupling constant of the ghost field is calculated from the ghost self-energy in a similar way as ζ_3^0 . At one loop, there is no diagram contributing to the ghost decoupling. At two loops, there is only one diagram per heavy flavour. We find

$$\begin{aligned} \tilde{\zeta}_3^0 = 1 + \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right)^2 \sum_{q=1}^{n_h} \left[-\frac{3}{64\epsilon^2} + \frac{1}{\epsilon} \left(\frac{5}{128} + \frac{3}{16} \log(m_q^0) \right) - \frac{89 + 6\pi^2}{768} \right. \\ \left. - \frac{5}{32} \log(m_q^0) - \frac{3}{8} \log^2(m_q^0) \right] \quad . \end{aligned} \quad (4.9)$$

The decoupling of the gluon-ghost vertex $\tilde{\zeta}_1^0$ is given by

$$\tilde{\zeta}_1^0 = 1 + \Gamma_{\bar{\eta}G\eta}^0(0,0) , \quad (4.10)$$

where $\Gamma_{\bar{\eta}G\eta}^0(p,p')$ is extracted from the 1PI amputated gluon-ghost Green function and p and p' are the incoming four-momenta of $\bar{\eta}$ and G respectively. Again, this term receives no contribution at one loop. At two loops, there are 5 non-massless diagrams for each heavy flavour. Two of them vanish because of colour, and the other three add up to zero. Therefore

$$\tilde{\zeta}_1^0 = 1 + \mathcal{O}(\alpha_s^3) . \quad (4.11)$$

Inserting Eqs. (4.11, 4.9, 4.7) into the relation (4.8) we find

$$\begin{aligned} \zeta_g^0 = 1 + \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right) & \left[-\frac{n_h}{12\epsilon} + \frac{L_q^0}{6} - \epsilon \left(\frac{L_{2,q}^0}{6} + n_h \frac{\pi^2}{144} \right) \right] \\ & + \left(\frac{\alpha_s^0 S_\epsilon}{\pi} \right)^2 \left\{ \frac{n_h^2}{96\epsilon^2} - \frac{n_h}{24\epsilon} \left[L_q^0 + \frac{3}{4} \right] + \frac{L_q^0}{8} + \frac{(L_q^0)^2}{24} + \frac{n_h}{24} \left[L_{2,q}^0 + \frac{11}{6} \right] + n_h^2 \frac{\pi^2}{576} \right\} . \end{aligned} \quad (4.12)$$

Here we introduced the notation

$$L_q^0 = \sum_{q=1}^{n_h} \log(m_q^0) \quad , \quad L_{2,q}^0 = \sum_{q=1}^{n_h} \log^2(m_q^0) . \quad (4.13)$$

We now renormalize the mass of the heavy quarks in the full theory according to Eq. (4.4). The mass renormalization constants in the full theory Z_{m_q} are obtained from the one- and two-loop corrections to the quark propagator [16]. We review here the main steps of this calculation. Let us denote the sum of all the one-particle irreducible (1PI) insertions into the quark propagator as $-i\Sigma_0(p)$,

$$-i\Sigma_0(p) = -i\Sigma_0^{1L}(p) - i\Sigma_0^{2L}(p) + \dots , \quad (4.14)$$

where $-i\Sigma_0^{1L}(p)$ is the sum of all the one-loop 1PI diagrams in the bare theory and so on. The full quark propagator then reads

$$\frac{i}{\not{p} - m_q^0 - \Sigma_0(p)} . \quad (4.15)$$

We can split $\Sigma_0(p)$ as

$$\Sigma_0(p) = \Sigma_{10}(p^2) + (\not{p} - m_q^0)\Sigma_{20}(p^2) ; \quad (4.16)$$

conversely, the quantities $\frac{1}{m_0}\Sigma_{10}$, Σ_{20} are extracted from the bare self-energy Σ_0 as

$$\frac{1}{m_0}\Sigma_{10} = \frac{1}{4}\text{Tr} \left(\frac{1}{m_0}\Sigma_0 + \frac{\not{p}}{p^2}\Sigma_0 \right) \quad , \quad \Sigma_{20} = \frac{1}{4p^2}\text{Tr} (\not{p}\Sigma_0) . \quad (4.17)$$

Combining Eqs. (4.15) and (4.16) we obtain

$$m_q^0 = m_q(\mu) \left[1 - \frac{1}{m_q}\Sigma_1^{1L} - \left(\frac{1}{m_q}\Sigma_1^{2L} + \frac{1}{m_q}\Sigma_1^{1L}\Sigma_2^{1L} \right) \right] . \quad (4.18)$$

The RHS of Eq. (4.18) is expressed in terms of renormalized quantities. On the other hand, the one-loop result for Σ_{10}^{1L} originally depends on the bare gauge parameter and on the bare strong coupling and is divided itself by the bare mass. Therefore, one also needs the one-loop renormalization of these three quantities in order to compute Z_{m_q} through two loops.

We compute the self-energy for arbitrary external momentum p , and set the mass of the heavy quarks to zero. In the $\overline{\text{MS}}$ scheme we recover the result [16]¹

$$Z_{m_q} = 1 - \frac{\alpha_s(\mu)}{\pi} \frac{1}{\epsilon} + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left[\frac{1}{\epsilon^2} \left(\frac{45 - 2n_f}{24} \right) + \frac{1}{\epsilon} \left(-\frac{101}{48} + \frac{5}{72}n_f \right) \right]. \quad (4.19)$$

This relation holds both in the full theory, where the number of active flavours n_f is $n_f = n_l + n_h$, and in the effective theory. In this case, $n_f = n_l$ and we need to replace the renormalized strong coupling in the full theory with the one of the effective theory.

So far, the coefficient for the mass decoupling and renormalization still depends on the renormalized strong coupling in the *full* theory. We decouple it using the relation [4]

$$\alpha'_s(\mu) = \frac{Z_\alpha(\zeta_g^0)^2}{Z'_\alpha} \alpha_s(\mu) = \zeta_g^2 \alpha_s(\mu). \quad (4.20)$$

The strong coupling renormalization constants are related to the coefficients of the β function as

$$Z'_\alpha = 1 - \frac{\alpha'_s(\mu)}{\pi} \frac{\beta'_0}{\epsilon} + \left(\frac{\alpha'_s(\mu)}{\pi} \right)^2 \left(\frac{\beta_0'^2}{\epsilon^2} - \frac{\beta'_1}{2\epsilon} \right). \quad (4.21)$$

Here β'_0 and β'_1 denote the first two coefficients of the β function in the light-flavours theory,

$$\beta'_0 = \frac{1}{4} \left(11 - \frac{2}{3}n_l \right), \quad \beta'_1 = \frac{1}{16} \left(102 - \frac{38}{3}n_l \right). \quad (4.22)$$

Combining Eqs. (4.4 -4.21) we get

$$m_q^0 = m_q(\mu) \left\{ 1 - \frac{\alpha'_s(\mu)}{\pi} \frac{1}{\epsilon} + \left(\frac{\alpha'_s(\mu)}{\pi} \right)^2 \left[\frac{45 - 2(n_h + n_l)}{24\epsilon^2} + \frac{48L_m + 10(n_h + n_l) - 303}{144\epsilon} \right] \right\}, \quad (4.23)$$

with

$$L_m = \sum_{q=1}^{n_h} \log \left(\frac{m_q(\mu)}{\mu} \right). \quad (4.24)$$

The next step is the renormalization of the bare strong coupling in the effective theory, $\alpha_s'^0$. The relevant renormalization constant is given in Eq. (4.21).

We finally renormalize the bare Wilson coefficient $C_1^0(\alpha'_s, m_q)$ using [3, 14, 15]

$$\begin{aligned} C_1 &= \frac{1}{1 + \alpha'_s(\mu) \frac{\partial}{\partial \alpha'_s(\mu)} \log Z'_\alpha} C_1^0 \\ &= \left[1 + \frac{\alpha'_s(\mu)}{\pi} \frac{\beta'_0}{\epsilon} + \left(\frac{\alpha'_s(\mu)}{\pi} \right)^2 \frac{\beta'_1}{\epsilon} \right] C_1^0. \end{aligned} \quad (4.25)$$

¹Note that in our conventions $d = 4 - 2\epsilon$, while in Ref. [16] $d = 4 + 2\epsilon$. This explains the sign difference in the $1/\epsilon$ terms.

Our final result for the renormalized Wilson coefficient reads

$$C_1 = -\frac{1}{3} \frac{\alpha'_s(\mu)}{\pi} \left\{ n_h + \frac{11}{4} \frac{\alpha'_s(\mu)}{\pi} n_h - \left(\frac{\alpha'_s(\mu)}{\pi} \right)^2 \left[-\frac{1877}{192} n_h + \frac{77}{576} n_h^2 + \frac{19L_m}{8} + n_l \left(\frac{67}{96} n_h + \frac{2L_m}{3} \right) \right] \right\}. \quad (4.26)$$

This is the main result of our paper. Note that the second term in the square brackets comes from diagrams containing two massive quarks loops. For $n_h = 1$ we recover the SM Wilson coefficient [9, 10, 17, 18] through order $\mathcal{O}(\alpha_s'^3)$.

At leading order in the heavy-quark expansion, and assuming a massless bottom quark, the gluon fusion cross-section and the decay width of the Higgs boson to gluons are proportional to the square of the Wilson coefficient. In this limit, their ratio with the corresponding quantities in the Standard Model are:

$$\frac{\sigma(gg \rightarrow H)^{(n_h)}}{\sigma(gg \rightarrow H)^{(SM)}} = \frac{\Gamma(H \rightarrow gg)^{(n_h)}}{\Gamma(H \rightarrow gg)^{(SM)}} = n_h^2 - \left(\frac{\alpha'_s(\mu)}{\pi} \right)^2 n_h \left[\frac{77}{288} n_h (n_h - 1) + \left(\frac{4}{3} n_l + \frac{19}{4} \right) \sum_q \log \left(\frac{m_q(\mu)}{m_t(\mu)} \right) \right] + \mathcal{O}(\alpha_s'^3). \quad (4.27)$$

where m_t the mass of the top-quark. The $\mathcal{O}(\alpha_s'^2)$ term in the above expression is generally very small. However, in a realistic phenomenological study [19, 20] the exact quark mass dependence of the cross-section as well as effects due to electroweak corrections need to be accounted for through NLO.

5. Numerical Results for gluon fusion cross section at the Tevatron

In this Section, we present our numerical results for the cross-section at the Tevatron, in a Standard Model with four generations. The Wilson coefficient for the fourth generation model is obtained by considering three heavy quarks in Eq. (4.26). We set the top-quark mass to

$$m_t = 170.9 \text{ GeV}.$$

For the fourth generation we consider two scenarios, corresponding to fourth generation down-quark masses of:

$$m_B = 300 \text{ GeV}, \quad m_B = 400 \text{ GeV}$$

and an up-quark mass given by

$$m_T - m_B = 50 \text{ GeV} + 10 \log \left(\frac{m_H}{115 \text{ GeV}} \right) \text{ GeV}. \quad (5.1)$$

This choice is permitted by constraints from electroweak precision tests, as described in Ref. [21].

The calculation of the total cross-section differs from the Standard Model only in the expression of the Wilson coefficient for the low energy effective Lagrangian. We combine

Eq. (4.26) with the known results for the Standard Model total cross-section at NNLO in the large top-mass limit of Refs [22–24].

Adopting the same approach as in Ref. [19], we first compute the ratio of the NNLO and LO cross-section in the effective theory. We estimate the contribution from Feynman diagrams with only top-quark and fourth generation quarks to the total cross-section, by multiplying this ratio with the exact leading order contributions of heavy quark diagrams in the full theory.

$$\sigma_{heavy}^{NNLO;(t,B,T)} \simeq \left(\frac{\sigma^{NNLO;(t,B,T)}}{\sigma^{LO;(t,B,T)}} \right)_{effective} \sigma_{exact}^{LO;(t,B,T)} \quad (5.2)$$

These contributions are enhanced by roughly a factor of 9 with respect to the corresponding Standard Model results, since

$$\sigma_{exact}^{LO;(t,B,T)} \simeq 9 \sigma_{exact}^{LO;(t)} \quad (5.3)$$

within a few percent.

Contributions from diagrams with bottom quark loops are small and we compute them exactly through the NLO order approximation [25, 26]. In comparison to their Standard Model counterparts, the most important interference terms of diagrams with bottom quarks only and diagrams with any of the heavier quarks are only enhanced by roughly a factor of three. Therefore, these contributions are suppressed by roughly a factor of $\sim 3/9$ in this model.

Finally, we include two-loop electroweak corrections from light-quark loops from the first two generations [27] in the complex mass scheme, and the corresponding three-loop mixed QCD and electroweak corrections as in Ref. [19]. These are enhanced by a factor of roughly 3 in comparison to the Standard Model, and are therefore suppressed by a factor of $1/3$ in this model. We ignore electroweak corrections with quarks from the third and fourth generation, which are already found to be very small in the Standard Model [28] for the Higgs mass range accessible at the Tevatron, when a complex mass scheme is employed.

In Table 1, we present the cross-section at a renormalization and factorization scale $\mu = \mu_f = \mu_r = m_H/2$ and estimate the scale variation error by varying the common scale μ in the interval $[\frac{m_H}{4}, m_H]$. We use the MSTW2008 NNLO parton distribution functions [29], and compute the uncertainty (with 90% CL) to the cross-section due to the parton distribution functions (including the parametric uncertainty of the value of α_s) according to Ref. [30].

The scale variation uncertainty and the uncertainty from the parton distributions are the dominant uncertainties. Essentially, they are the same as in the Standard Model cross-section with only three generations. We note that Ref. [19] preceded the release of Ref. [30] where it became possible to include the uncertainty of the α_s value in the fitted parton densities and the corresponding parton density uncertainty was estimated to be smaller.

6. Conclusions

In this paper, we constructed an effective field theory for extensions of the Standard Model with many heavy quarks coupling to the Higgs boson via top-like Yukawa interactions. We

have computed the required Wilson coefficient of the $-\frac{H}{4v}G_{\mu\nu}G^{\mu\nu}$ operator through NNLO in the strong coupling expansion. We found the result

$$C_1 = -\frac{1}{3}\frac{\alpha'_s(\mu)}{\pi}\left\{n_h + \frac{11}{4}\frac{\alpha'_s(\mu)}{\pi}n_h - \left(\frac{\alpha'_s(\mu)}{\pi}\right)^2\left[-\frac{1877}{192}n_h + \frac{77}{576}n_h^2 + \frac{19L_m}{8} + n_l\left(\frac{67}{96}n_h + \frac{2L_m}{3}\right)\right]\right\}. \quad (6.1)$$

This result can be utilized by the ongoing experimental studies at the Tevatron and the LHC to constrain such models. The cross-section and the decay width are enhanced by roughly the square of the number of heavy quarks with respect to the Standard Model. The Tevatron has put stringent limits on the Standard Model Higgs boson gluon fusion cross-section [1]. Equivalent studies can be performed in models with additional quarks. We have presented numerical results for the gluon fusion cross-section at the Tevatron in a non-minimal Standard Model with four generations. The theoretical uncertainties of the cross-section are practically independent of the number of heavy quarks and very similar to the Standard Model.

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$m_H(\text{GeV})$	$\sigma_{(1)}(fb)$	$\sigma_{(2)}(fb)$	$\frac{\delta\sigma}{\sigma}(\text{pdf} + \alpha_s)\%$	$\frac{\delta\sigma}{\sigma}(\text{scale})\%$
110	12384	12308	+12% , -11%	+12% , -8%
115	10798	10725	+12% , -11%	+12% , -8%
120	9449.9	9384.3	+12% , -11%	+12% , -8%
125	8298.8	8240.0	+12% , -12%	+12% , -8%
130	7314.0	7258.7	+12% , -12%	+12% , -8%
135	6465.1	6414.2	+12% , -12%	+12% , -8%
140	5731.4	5684.1	+13% , -12%	+12% , -8%
145	5094.6	5050.4	+13% , -12%	+12% , -8%
150	4540.5	4498.5	+13% , -12%	+12% , -8%
155	4055.6	4017.6	+13% , -12%	+12% , -8%
160	3630.2	3595.1	+13% , -13%	+12% , -8%
165	3253.7	3220.7	+14% , -13%	+12% , -8%
170	2924.1	2893.2	+14% , -13%	+12% , -8%
175	2633.9	2604.4	+14% , -13%	+12% , -8%
180	2376.7	2348.9	+14% , -13%	+12% , -8%
185	2147.2	2121.5	+15% , -13%	+12% , -8%
190	1943.9	1919.7	+15% , -14%	+12% , -8%
195	1763.2	1740.2	+15% , -14%	+12% , -8%
200	1601.8	1580.0	+15% , -14%	+12% , -8%
205	1457.5	1436.7	+16% , -14%	+12% , -8%
210	1328.1	1308.4	+16% , -14%	+12% , -8%
215	1212.0	1193.2	+16% , -14%	+12% , -8%
220	1107.7	1089.6	+16% , -15%	+12% , -8%
225	1013.6	996.33	+17% , -15%	+12% , -8%
230	928.61	912.21	+17% , -15%	+12% , -8%
235	852.00	836.33	+17% , -15%	+12% , -8%
240	782.52	767.44	+17% , -15%	+12% , -8%
245	719.64	705.19	+18% , -15%	+12% , -8%
250	662.60	648.81	+18% , -16%	+12% , -8%
255	610.74	597.51	+18% , -16%	+12% , -8%
260	563.53	550.90	+19% , -16%	+12% , -8%
265	520.60	508.52	+19% , -16%	+12% , -8%
270	481.49	469.93	+19% , -16%	+12% , -8%
275	445.86	434.72	+20% , -16%	+12% , -8%
280	413.24	402.68	+20% , -17%	+12% , -8%
285	383.56	373.28	+20% , -17%	+12% , -8%
290	356.39	346.53	+21% , -17%	+12% , -8%
295	331.53	322.04	+21% , -17%	+12% , -8%
300	308.70	299.71	+21% , -17%	+12% , -8%

Table 1: The NNLO cross-section for Higgs production via gluon fusion at the TEVATRON. $\sigma_{(1)}$ corresponds to $m_B = 300\text{GeV}$ and $\sigma_{(2)}$ to $m_B = 400\text{GeV}$. The mass of the fourth generation up quark is given by Eq. (5.1)